Introduction: Tests of Significance on distributions of counts:

Chi-Square Test for Goodness of Fit

(Do test results support Mendel's genetic principles?)

Chi-Square Test for Independence

(Was surviving the Titanic sinking independent of a passenger's status?)

Chi-Square Test Homogeneity of Proportions

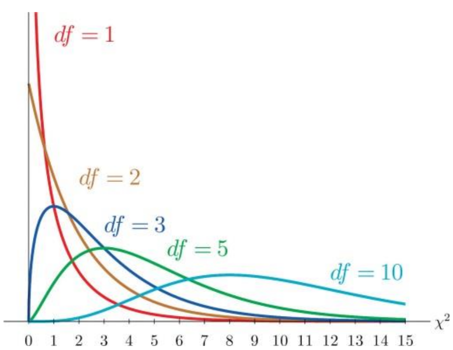
(Do students, teachers, and staff show the same distributions in types of cars driven?)

Chi-Square Tests:

- These are tests about distributions or relationships involving ____________________ variables.
- The alternative hypothesis is always "the null hypothesis is not correct", so H_a is ______________.
- Chi-Square test statistics are always calculated using ____________.
- Formula:

(measures how far the observed counts are from the expected counts)

<u>Chi-Square Distributions</u>:



A family of distributions that take only ____________ values and are skewed to the __________ A particular chi-square distribution is specified by giving its degrees of freedom.

As the degrees of freedom increase, the density curve becomes less skewed (closer to _____________), and larger values become more probable.

The __________ of a particular chi-square distribution is __________ its degrees of freedom.

For df > 2, the mode (peak) of the χ^2 density curve is at df – 2.

Q&A:

1. A random sample of mice is obtained, an each mouse is timed as it moves through a maze to a treat at the end. After several days of training, each mouse is timed again. The data should be analyzed using:
   A) A z-test of proportions B) a two-sample test of means C) a paired t-test D) a chi-square test

   Why:

2. True or False.

   Unless you have counts, you cannot use χ^2 methods __________

   With χ^2, the alternative hypothesis can be one-sided or two-sided. ___________

   With df = 3, bigger values of χ^2 result in greater are under the curve and mode is 2. ____________

3. In a study about TV audience at the 6 p.m. time slot, the following null hypothesis was formulated.
H_o: TV audience is distributed over channels 4, 6, and 10 with percentages 30%, 45%, and 25%, respectively.
What would be an appropriate alternative hypothesis?
H_a:

Example: Finding Expected Values and $\chi^{2.}$

A Fair Die?

Jenny made a six-sided die in her ceramics class and rolled it 60 times to test if each side was equally likely to show up on top. (See results below) Assuming that her die is fair,

Outcome	Observed	Expected
1	13	
2	11	
3	6	
4	12	
5	10	
6	8	
Total		

a) Calculate the expected counts for each side.

b) Calculate the value of the chi-square statistic. Show work.

c) Using the appropriate degrees of freedom, calculate the P-value. What conclusion can you make about Jenny's die?

CYU: Pg.681, 684 (Complete on your own paper and attach to lesson)

	The Chi-Square Goodness-of-Fit Test
STATE	State your hypotheses and significance level α (0.05 if none is given) H_o: The specified distribution of the categorical variable is correct H_a: The specified distribution of the categorical variable is not correct (Remember hypotheses are always about a population, never about the sample)
PLAN	Find the expected counts values for each category assuming H_o is true. Check conditions: 1) RANDOM: The data come from a random sample or a randomized experiment. 2) LARGE SAMPLE SIZE: All expected counts are at least 5. 3) INDEPENDENT: Individual observations are independent. When sampling without replacement, check that the population is at least 10 times as large as the sample (the 10% condition)
DO	Calculate the chi-square statistic (Formula or Calculator) P-value is the area to the right of χ^2 under the density curve of the chi-square distribution with degrees of freedom = categories – 1

CONCLUDE	If p-value < α, we reject the H_o. We have sufficient evidence to conclude H_a (in context) If p-value $\geq \alpha$, we fail to reject the H_o. We don't have sufficient evidence to conclude H_a (in context)

Example: When Were You Born?
Are births evenly distributed across the days of the week? The one-way table below shows the distribution of births across the days of the week in a random sample of 140 births from local records in a large city:

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
Births	13	23	24	20	27	18	15

Do these data give significant evidence that local births are not equally likely on all days of the week?

CYU: Pg.689 (Complete on your own paper and attach to lesson)

Free Response Practice:

According to the 2000 census, of all U.S. residents aged 20 and older, 19.1% are in their 20s, 21.5% are in their 30s, 21.1% are in their 40s, 15.5% are in their 50s, and 22.8% are 60 and older. The table below shows the age distribution for a sample of U.S. residents aged 20 and older. Members of the sample were chosen by randomly dialing landline telephone numbers.

Category	Count
20-29	141
30-39	186
40-49	224
50-59	211
60+	286
Total	1048

Do these data provide convincing evidence that the age distribution of people who answer landline telephone surveys is not the same as the age distribution of all U.S. residents?

Describe a type II error in this context.

What if we want to compare the distributions of a single categorical variable across several populations or treatments?

Example: Saint-John's-Wort and depression
An article in the *Journal of the American Medical Association* (April 10, 2002, vol 287, no 14) reports the results of a study designed to see if the herb, Saint-John's-Wort, is effective in treating moderately severe cases of depression. The study involved 338 subjects who were being treated for major depression. The subjects were randomly assigned to receive one of three treatments: St. John's Wort (an herb), Zoloft (a prescription drug) or placebo for an 8-week period. The table below summarizes the results of the experiment.

	St. John's Wort	Zoloft	Placebo	Total
Full Response	27	27	37	91
Partial Response	16	26	13	55
No Response	70	56	66	192
Total	113	109	116	338

Problem:

(a) Calculate the conditional distribution (in proportions) of the type of response for each treatment.

(b) Make an appropriate graph for comparing the conditional distributions in part (a).

(c) Compare the distributions of response for each treatment.

CYU: Pg.698

Multiple Comparisons – Statistical methods for dealing with multiple comparisons usually have 2 parts:

1) An overall test to see if there is good evidence of any differences among the parameters that we want to compare. (Uses chi-square statistic and distributions)
2) A detailed follow up analysis to decide which of the parameters differ and to estimate how large the differences are.

See Example: Does Background Music Influence What Customers Buy? (pg.700)

Formula: Finding Expected Counts:

Example: Saint-John's-wort and depression

Here is a summary of the results of the experiment comparing the effects of St. John's wort, Zoloft, and a placebo.

	Saint-John's-wort	Zoloft	Placebo	Total
Full Response	27	27	37	91
Partial Response	16	26	13	55
No Response	70	56	66	192
Total	113	109	116	338

Problem:
a) Calculate the expected counts for the three treatments, assuming that all three treatments are equally effective.

b) Calculate the chi-square statistic. Show work.
Formula for Chi-Square:

See Example: Does Background Music Influence What Customers Buy? (pg. 702)

CYU: Pg.703

Chi-Square Test for Homogeneity
State your hypotheses: H_o: There is no difference in the distribution of a categorical variable for several populations or treatments. H_a: There is a difference in the distribution of a categorical variable for several populations or treatments. Find the expected counts: Check conditions: • RANDOM: The data come from separate random samples from each population of interest or from the groups in a randomized experiment. • LARGE SAMPLE SIZE: All expected counts are at least 5. • INDEPENDENT: Both the samples or groups themselves and the individual observations in each sample or group are independent. When sampling without replacement, check that the individual populations are at least 10 times as large as the samples (10% condition) Calculate the chi-square statistic. Determine degrees of freedom and P-value.   If H_o is true, the chi-square statistic has approximately a chi-square distribution with: degrees of freedom = (number of rows – 1)(number of columns – 1) The P-value is the area to the right of chi-square under the corresponding density curve. Conclusion: If P-value < α, we reject H_o. There is sufficient evidence to conclude H_a. If P-value > α, we fail to reject H_o. There is not sufficient evidence to conclude H_a } IN CONTEXT

See Example: Does Background Music Influence What Customers Buy? (Pg. 704)

Example: Saint-John's-Wort and depression

Earlier we started a significance test of

H_0: There is no difference in the distribution of responses for patients with moderately severe cases of depression when taking Saint-John's-wort, Zoloft, or a placebo.

H_a: There is a difference in the distribution of responses for patients with moderately severe cases of depression when taking Saint-John's-wort, Zoloft, or a placebo.

The value of χ^2 was 8.72.

Problem:

(a) Verify that the conditions for this test are satisfied.

(b) Calculate the *P*-value for this test.

(c) Interpret the *P*-value in context.

(d) What is your conclusion?

CYU: Pg.705

Example: Are cell-only telephone users different? (The chi-square test for homogeneity)
Random digit dialing telephone surveys used to exclude cell phone numbers. If the opinions of people who have only cell phones differ from those of people who have landline service, the poll results may not represent the entire adult population. The Pew Research Center interviewed separate random samples of cell-only and landline telephone users who were less than 30 years old. Here's what the Pew survey found about how these people describe their political party affiliation.

	Cell-only sample	Landline sample
Democrat or lean Democratic	49	47
Refuse to lean either way	15	27
Republican or lean Republican	32	30
Total		

a) Construct an appropriate graph to compare the distributions of political party affiliation for cell-only and landline phone users.

b) Do these data provide convincing evidence that the distribution of party affiliation differs in the cell-only and landline populations? Carry out a significance test at the $\alpha = 0.05$ level

CYU: Pg.708

Comparing several proportions:

Example: Cocaine Addiction is Hard to Break.
Cocaine addicts need cocaine to feel any pleasure, so perhaps giving them an antidepressant drug will help. A three-year study with 72 chronic cocaine users compared an antidepressant drug called desipramine with lithium (a standard drug to treat cocaine addiction) and a placebo. One-third of the subjects were randomly assigned to receive each treatment. Here are the results:

		Cocaine Relapse?	
Group	Treatment	Yes	No
1	Desipramine	10	14
2	Lithium	18	6
3	Placebo	20	4

a) Make a graph to compare the rates of cocaine relapse for the three treatments. Describe what you see.

b) Are the differences between the three groups statistically significant at the 1% level? Give appropriate evidence to support your answer.

CYU: Pg. 713

See Example: Do Angry People Have More Heart Disease? (pgs. 713 and 715)

Chi-Square Test for Association/Independence
State your hypotheses: H_o: There is no association between two categorical variables in the population of interest. H_a: There is an association between two categorical variables in the population of interest. or H_o: Two categorical variables are independent in the population of interest. H_a: Two categorical variables are not independent in the population of interest. Find the expected counts Check conditions: • RANDOM: The data come from separate random samples or randomized experiment. • LARGE SAMPLE SIZE: All expected counts are at least 5. • INDEPENDENT: Individual observations are independent. When sampling without replacement, check that the individual populations are at least 10 times as large as the samples (10% condition) Calculate the chi-square statistic, degrees of freedom, and P-value. If H_o is true, the chi-square statistic has approximately a chi-square distribution with: degrees of freedom = (number of rows – 1)(number of columns – 1) The P-value is the area to the right of chi-square under the corresponding density curve. Conclusions: Conclusion: If P-value < α, we reject H_o. There is sufficient evidence to conclude H_a. } IN CONTEXT If P-value > α, we fail to reject H_o. There is not sufficient evidence to conclude H_a

Example: Do Angry People Have More Heart Disease? (Chi-Square test for association/independence)
Here is the complete table of observed and expected counts for the CHD and anger study side by side:

	Observed			Expected		
	Low	Moderate	High	Low	Moderate	High
CHD	53	110	27	69.73	106.08	14.19
No CHD	3057	4621	606	3040.27	4624.92	618.81

Do the data provide convincing evidence of an association between anger level and heart disease in the population of interest? Carry out an appropriate test to help answer this question.

CYU: Pg.718